GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 - JANUARY 2016

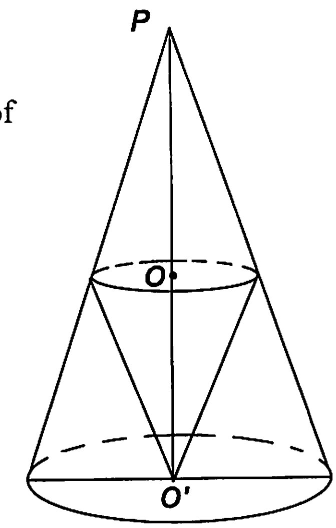
ROUND 1 - Volume and Surface Area of Solids

1. ______________ : ______________

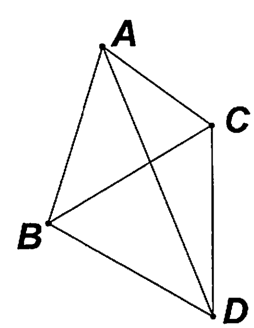
2. ______________________________

3. ______________________________

1. The diagram at the right depicts right circular cones. If $PO : OO' = 2:3$, compute the ratio of the sum of the volumes of the two smaller cones sharing the circular cross section O to the volume of the largest cone.



2. $ABCD$ is a regular tetrahedron (pyramid with a triangular base) with all edges 2 inches in length. If L and M are midpoints of $\overline{BC}$ and $\overline{AD}$ respectively, then compute LM (in inches).



3. The frustum of a cone has a volume of 1872π. The radii of its bases and its height are all integers. If $r_2 : r_1 = 5:2$ and $h = r_1 + 1$. Compute the lateral surface area of this frustum.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 - JANUARY 2016

ROUND 2 - Inequalities and Absolute Value

1. ______________________

2. ______________________

3. ______________________

1. For what value(s) of x is the following statement true?

   $$|x+1|^2 - 5|x+1| + 6 = 0$$

2. Solve over the reals: $|7-2x| + 5x \geq 3x + 3$

3. Given: $\dfrac{(x-4)^2(x+3)(x-3)}{3(x-3)^2(x+3)} \leq 0$

   The solution set is $\{x \mid \_\_\_\_\_\_\_\_\_\_\_\_\}$

   Fill in a simplified expression for the condition all solutions must satisfy.

   You may use the words "and" and "or" and the comma.

   You may **not** use interval notation.

GREATER BOSTON MATHEMATICS LEAGUE

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ROUND 3 - Similar Polygons, Circles and Area

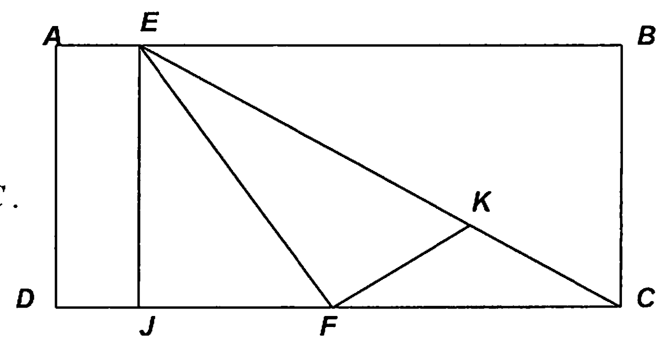
1. ______________________

2. ______________________

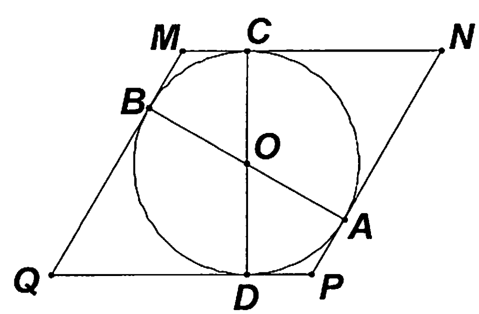
3. ________ : ________ : ________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.

1. Given: Rectangle $ABCD$ with
   $AD = 16$, $AE = 4$, $\overline{EJ} \parallel \overline{AD}$ and
   $DJ : JF : FC = 1:2:3$.
   If $EK : KC = 3:1$, compute the area of ΔKFC.



2. In circle O, with integer radius r, parallel chords $\overline{PQ}$ and $\overline{RS}$ are <u>equidistant</u> from the center O. Let $a = \max(PQ, RS)$, and b, the distance between the chords, be <u>integers</u>. Compute the <u>minimum</u> numerical value of $a + b$.



3. Circle O is inscribed in rhombus $MNPQ$.
   $m(\angle NPQ) : m(\angle OAN) = 4:3$
   $\overline{AB}$ and $\overline{CD}$ are diameters of circle O.
   The area of the rhombus is $72\sqrt{3}$.
   Compute the ratio $MC : NA : OD$, where MC is an integer.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 - JANUARY 2016

ROUND 4 - Sequences and Complex Numbers

1. ______________________________

2. ((_____ , _____ , _____), _____)

3. ______________________________

1. If the second term in an arithmetic sequence is $11-5i$ and the fifth term is $2-11i$, determine the sum of the first 20 terms in this sequence in $a+bi$ form.

2. Given the sequence $S=(3,-5,7),(7,-5,3),(11,-5,-1),(15,-5,-5),\ldots$.
   If W denotes the 29th term in this sequence, and T denotes the sum of all the numbers in the first 29 ordered triples, compute the ordered pair (W,T).

3. Consider the infinite series $S=2i^3+6i^8+10i^{13}+14i^{18}+18i^{23}+\ldots$.
   Compute the partial sum S_{22}, the sum of the first 22 terms.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 - JANUARY 2016

ROUND 5 - Conics

1. (________ , ________ , ________)

2. (____________ , ____________)

3. V_1(_____ , _____) V_2(_____ , _____)

1. Given: $P(-5, 2)$ and $Q(3, 8)$
   $\overline{PQ}$ is a diameter of a circle defined by $(x-h)^2+(y-k)^2=r^2$.
   Compute the ordered triple (h, k, r).

2. The parabola $y=ax^2+b$ opens <u>up</u>, has its vertex at an endpoint of the minor axis of the hyperbola $x^2-y^2=1$, and passes through both foci of this hyperbola.
   Compute the ordered pair (a,b).

3. Compute the coordinates of the vertices of the major axis of the ellipse defined by $9x^2+8y^2-84y+108=0$. [The order in which they are listed is irrelevant.]

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 - JANUARY 2016

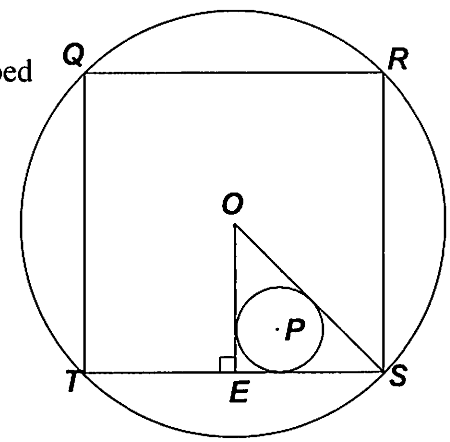
TEAM ROUND

3 pts. 1. ______________________

3 pts. 2. ______________________

4 pts. 3. ______________________

1. *QRST* is a square. Let A denote the area of the circumscribed circle O. Let B denote the area of the circle P, inscribed in right ΔOES. If $A = kB$, compute k in simplified radical form.



2. For what real values of x is the following statement true?

$$\frac{x+2}{(x+1)(x-3)}+\frac{x-1}{(x-2)(x-3)}+\frac{x+3}{(x+1)(x-2)}+\frac{x^2(x-4)+(x+8)}{(x+1)(x-2)(x-3)}\geq 0$$

3. $(t_1, t_2, t_3, t_4, t_5, t_6, t_7)$ denotes the first seven terms of the arithmetic sequence

$$\frac{z+1}{2},\ 4w-5,\ 3y-4,\ 4x+3,\ 7x-6,\ x+z+3y,\ 5y+6,\ \ldots\ .$$

Compute the ordered quadruple (t_1, t_2, t_6, t_7).

GREATER BOSTON MATHEMATICS LEAGUE

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Answer Sheet

Round 1

1. $4:25$
2. $\sqrt{2}$
3. 420π

Round 2

1. $1,-3,2,-4$ (in any order)
2. all reals
3. $x<3$ and $x\neq-3$ or $x=4$
   or equivalent, as discussed in solution key

Round 3

1. 24
2. $1:3:\sqrt{3}$
3. $61:140$

Round 4

1. $-290-440i$
2. $((115,-5,-105),145)$
3. $46-42i$

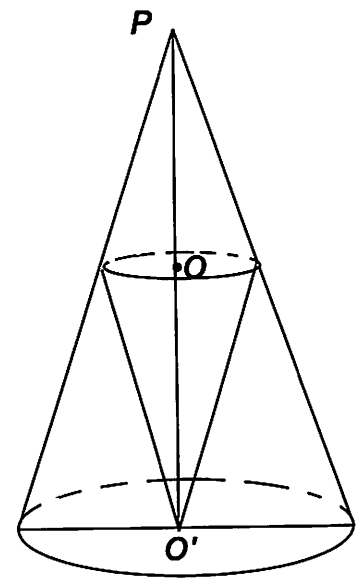
Round 5

1. $(-1, 5, 5)$
2. $\left(\frac{1}{2},-1\right)$
3. $(0,9),\left(0,\frac{3}{2}\right)$

Team Round

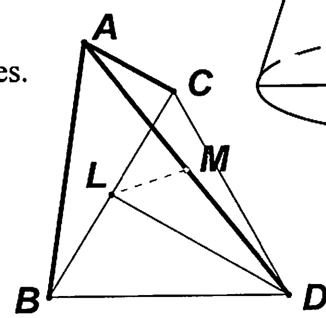
1. $4\left(3+2\sqrt{2}\right)$ (3 pts)
2. $-1<x<3$ and $x\neq2$ (3 pts)
   (or equivalent, as discussed in the solution key)
3. $(5,11,35,41)$ (4 pts)

Detailed Solutions for GBML Meet 4 - JANUARY 2016

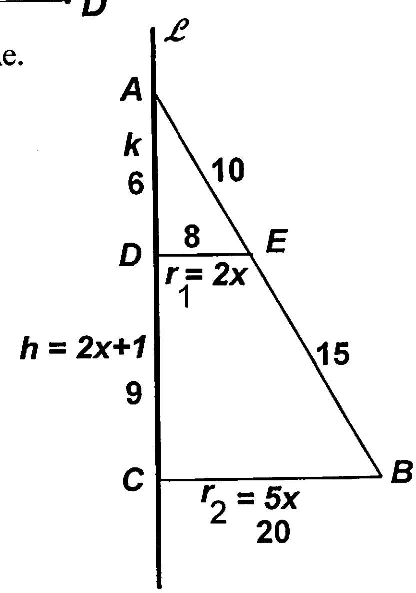


ROUND 1

1. Let $h_1 = PO = 2t$, $h_2 = OO' = 3t$, and $h_3 = PO' = 5t$.
   By similar triangles, the base radii must be in a 2 : 5 ratio.
   Thus, $\dfrac{V_1+V_2}{V_3} = \dfrac{\frac{1}{3}\pi(2r)^2(2t)+\frac{1}{3}\pi(2r)^2(3t)}{\frac{1}{3}\pi(5r)^2(5t)} = \dfrac{\frac{1}{3}\pi(2r)^2(5t)}{\frac{1}{3}\pi(5r)^2(5t)} = \underline{\dfrac{4}{25}}$.



2. All 4 faces of the tetrahedron are equilateral triangles.
   $LC = 1, CD = 2 \Rightarrow DL = \sqrt{3}$
   $\overline{LM} \perp \overline{AD} \Rightarrow LM^2 + MD^2 = LD^2$
   $\Rightarrow LM^2 = \left(\sqrt{3}\right)^2 - 1^2 = 2 \Rightarrow LM = \underline{\sqrt{2}}$.



3. Rotating right ΔABC around vertical line $\mathcal{L}$ produces a small cone, a large cone and the frustum of the larger cone.
   $r_2 : r_1 = 5:2 \Rightarrow r_1 = 2x,\ r_2 = 5x$. By similar triangles,
   $\dfrac{AD}{AC} = \dfrac{DE}{CB} \Rightarrow \dfrac{k}{h+k} = \dfrac{k}{k+2x+1} = \dfrac{2}{5} \Rightarrow k = \dfrac{2}{3}(2x+1)$.
   $V(\text{frustum}) = V(\text{cone}_2) - V(\text{cone}_1) =$
   $\dfrac{\pi}{3}(5x)^2\left(\dfrac{5}{3}(2x+1)\right) - \dfrac{\pi}{3}(2x)^2\dfrac{2}{3}(2x+1) = 1872\pi$
   $\Leftrightarrow \dfrac{1}{9}x^2(2x+1)(125-8) = 13x^2(2x+1) = 1872$
   $\Leftrightarrow x^2(2x+1) = \dfrac{1872}{13} = 144 = 16\cdot 9 \Rightarrow x = 4,\ h = 9,\ k = 6$.
   Noting the triangles similar to the 3-4-5 right triangle, we have $(AE, AB, CB) = (10, 25, 20)$, as indicated.
   Using the lateral surface area formula $A = \pi rl$, we have $\pi\cdot 20\cdot 25 - \pi\cdot 8\cdot 10 = \underline{\mathbf{420\pi}}$.

   Alternatively, using the formulas for a frustum $\left[V = \pi\dfrac{h}{3}\left(r^2+rs+s^2\right)\right]$ and $LSA = \pi(r+s)l$,

   where l denotes the slant height. Calling $(r_1, r_2, h) = (2x, 5x, 2x+1)$, we have (omitting π),

   $\dfrac{2x+1}{3}\left(4x^2+10x^2+25x^2\right) = 1872 \Rightarrow (2x+1)\left(39x^2\right) = 3(1872) \Rightarrow x^2(2x+1) = 144 = 16\cdot 9$.
   By inspection, $x = 4$ and the base radii are 8 and 20 and the height is 9. The slant height is the hypotenuse of a right triangle with legs of 9 and $(20-8) = 12$, or 15 (A cute shortcut!).
   Thus, the lateral surface area of the frustum is $\pi(8+20)15 = \underline{\mathbf{420\pi}}$.

Detailed Solutions for GBML Meet 4 - JANUARY 2016

ROUND 2

1. $|x+1|^2-5|x+1|+6=0 \Leftrightarrow (|x+1|-2)(|x+1|-3)=0$

   $\Rightarrow |x+1|=2$ or $|x+1|=3 \Rightarrow x=-1\pm 2$ or $-1\pm 3 \Rightarrow$ $x=$ **<u>1,−3,2,−4</u>**.

2. $|7-2x|+5x \geq 3x+3 \Leftrightarrow |7-2x| \geq 3-2x \Leftrightarrow \begin{cases} 7-2x \geq 3-2x & \text{if } 7-2x \geq 0 \\ 2x-7 \geq 3-2x & \text{if } 7-2x<0 \end{cases}$.

   Case 1 $\left(x \leq \frac{7}{2}\right)$: $7-2x \geq 3-2x \Leftrightarrow 4 \geq 0$ which is true for all x.

   Thus, any value of $x \leq \frac{7}{2}$ satisfies the original inequality.

   Case 2 $\left(x > \frac{7}{2}\right)$: $2x-7 \geq 3-2x \Leftrightarrow 4x \geq 10 \Rightarrow x \geq \frac{5}{2}$.

   Which of these values overlap the domain of definition? Only $x > \frac{7}{2}$

   Combining the two cases, we have the solution set is **<u>all reals</u>**.

3. Provided $x \neq \pm 3$,

   the solution set of $\frac{(x-4)^2(x+3)(x-3)}{3(x-3)^2(x+3)} \leq 0$ is equivalent to the solution set of $\frac{(x-4)^2}{(x-3)} \leq 0$.

   Provided $x \neq 4$, the solution set of $\frac{(x-4)^2}{(x-3)} \leq 0$ is equivalent to the solution set of $\frac{1}{x-3} \leq 0$.

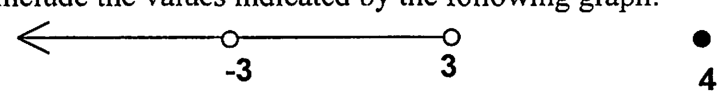
   The solution set of this inequality is $x<3$.

   Has any value been excluded which satisfies the original inequality? Yes, $x=4$.

   Has any value been included which does **<u>not</u>** satisfy the original inequality? Yes, $x=-3$.

   Thus, there are many ways to express the solution set.

   They all must include the values indicated by the following graph:



   Acceptable Solutions:

   $x<3$ and $x \neq -3$ or $x=4$ (since "and"s are evaluated before "or"s)

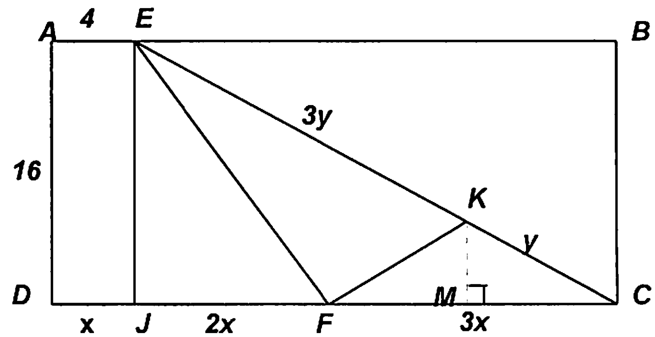
   $(x<3$ and $x \neq -3)$, $x=4$

   $x<3, -3<x<3, x=4$ (commas may be replaced with "or"s)

   Unacceptable: $(-\infty,-3),(-3,3), x=4$ is not allowed.

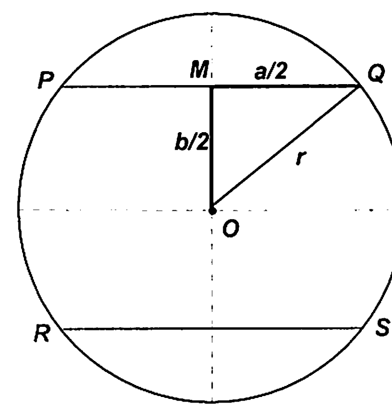
Detailed Solutions for GBML Meet 4 - JANUARY 2016

ROUND 3

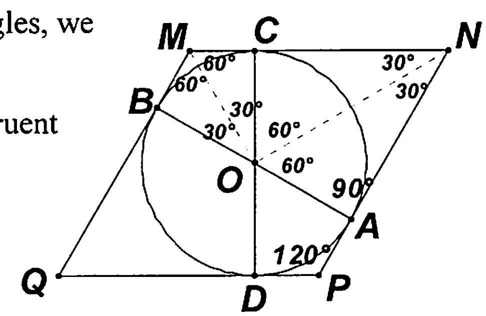


1. Clearly, $x = 4 \Rightarrow FC = 12$.
   Drop a perpendicular from K to $\overline{DC}$.
   $\Delta KMC \sim \Delta EJC \Rightarrow$
   $\frac{KM}{EJ} = \frac{KC}{EC} \Leftrightarrow \frac{KM}{16} = \frac{y}{4y} = \frac{1}{4} \Rightarrow KM = 4$.
   Therefore, the area is $\frac{1}{2} \cdot 4 \cdot 12 = \underline{24}$.

2. Since chords $\overline{PQ}$ and $\overline{RS}$ are equidistant from the center O, they must have the same length.
   Clearly, $MQ = \frac{a}{2}$ and $MO = \frac{b}{2}$. Applying the Pythagorean Theorem to ΔOMQ, we have $r^2 = \frac{a^2 + b^2}{4}$ and $r = \frac{1}{2}\sqrt{a^2 + b^2}$.
   To insure that r is an integer,
   $a^2 + b^2$ must be an <u>even</u> perfect square.
   Considering multiples of the 3-4-5 right triangle,
   $6^2 + 8^2 = 36 + 64 = 100$ and $r = 5$ for the minimum sum of
   $a + b = 6 + 8 = \underline{14}$.



3. Since a radius drawn to a point of tangency forms right angles, we have $m(\square\ OAN) = 3a = 90 \Rightarrow a = 30 \Rightarrow m(\square\ NPQ) = 120°$.
   Since $\overline{AB}$ and $\overline{CD}$ divide the rhombus into a pair of congruent kites and the diagonals of a rhombus are perpendicular, we notice that each of the kites is comprised of 30-60-90 right triangles. Obviously, the rhombus can be scaled to have any area, without changing the requested ratios, so without loss of generality,
   let $MC = 1$. Then: $OC = \sqrt{3},\ CN = NA = \sqrt{3} \cdot \sqrt{3} = 3$.
   Thus, the required ratio is $MC : NA : OD = \underline{\mathbf{1 : 3 : \sqrt{3}}}$.



Detailed Solutions for GBML Meet 4 - JANUARY 2016

ROUND 4

1. $\begin{cases} t_2 = a+d = 11-5i \\ t_5 = a+4d = 2-11i \end{cases} \Rightarrow (a,d) = (14-3i, -3-2i)$

   $S_n = \frac{n}{2}\left(2a+(n-1)d\right) \Rightarrow S_{20} = \frac{20}{2}\left(28-6i+19(-3-2i)\right) = \underline{\mathbf{-290-440}\boldsymbol{i}}$.

2. Given $S = (3,-5,7), (7,-5,3), (11,-5,-1), (15,-5,-5), ...$

   The x-coordinates are increasing by 4, the z-coordinates are decreasing by 4, while the y-coordinates remain unchanged. Thus, each ordered triple has the form $(3+4n,-5,7-4n)$.

   $n = 28$ produces t_{29}, namely $(3+28\cdot 4,-5,7-28\cdot 4) = (115,-5,-105)$

   The 3 numbers in every ordered triple sum to 5.

   Therefore, $(W,T) = \underline{\mathbf{((115,-5,-105),145)}}$

3. $S = (t_1+t_2+t_3+t_4)+(t_5+t_6+t_7+t_8)+... = (-2i+6+10i-14)+(-18i+22+26i-30)+...$.

   The terms t_n alternate between purely imaginary and purely real.

   Consider the sequence of purely real terms. $R = (6-14)+(22-30)+(38-46)+...$

   R consists of one pair of terms from each parenthesized group of 4 terms in S.

   We have 5 pairs of purely real terms in the first 20 terms of the S-series.

   Since each pair $(t_2+t_4, t_6+t_8, ...)$ sums to -8, the real part of $S_{22} = 5(-8)+t_{22}$.

   Since the first terms in each of the parenthesized terms of R form an arithmetic sequence, $t_{22} = 6+5(16) = 86$. Thus, the real part of S_{22} is 46.

   Consider the purely imaginary terms. $I = (-2i+10i)+(-18i+26i)+(-34i+42i)+...$

   I consists of one pair of terms from each parenthesized group of 4 terms in S.

   We have 5 pairs of purely imaginary terms in the first 20 terms of the S-series.

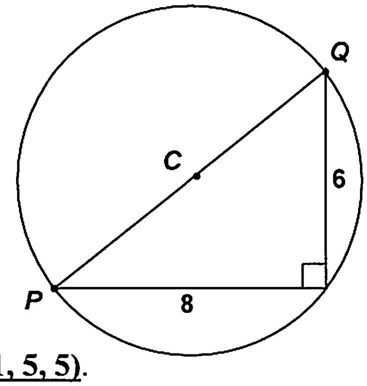
   Since each pair $(t_1+t_3, t_5+t_7, ...)$ sums to $8i$, the imaginary part of $S_{22} = 5(8i)+t_{21}$.

   Since the first terms in each of the parenthesized terms of I form an arithmetic sequence, $t_{21} = -2i+5(-16i) = -82i$. Thus, the imaginary part of S_{22} is $-42i$.

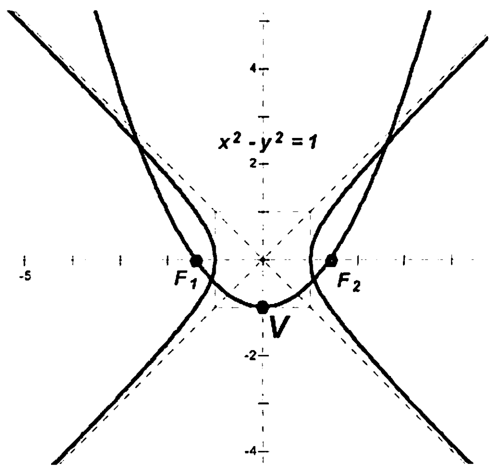
   Therefore, $S_{22} = \underline{\mathbf{46-42}\boldsymbol{i}}$.

Detailed Solutions for GBML Meet 4 - JANUARY 2016

ROUND 5



1. If $P(-5, 2)$ and $Q(3, 8)$, the center of the circle is the midpoint of $\overline{PQ}$, namely, $\left(\frac{-5+3}{2}, \frac{2+8}{2}\right) = C(-1,5)$ and $\overline{PQ}$ is a diameter.
The radius of the circle is CQ (or CP). Between points P and Q, the change in x is 8; the change in y is 6. We recognize the Pythagorean Triple (6, 8, 10). $PQ = 10 \Rightarrow r = 5$. Thus, $(h, k, r) =$ <u>$(-1, 5, 5)$</u>.



2. $x^2 - y^2 = 1 \Rightarrow a = b = 1 \Rightarrow c = \sqrt{2}$. The hyperbola is origin-centered, opens left-right, and has foci at $\left(\pm\sqrt{2}, 0\right)$. The endpoints of the minor axis are $(0, \pm 1)$ and the parabola must have its vertex at $(0, -1)$. Substituting in $y = ax^2 + b$,
we have $\begin{cases} -1 = a(0)^2 + b \\ 0 = a\left(\pm\sqrt{2}\right)^2 - 1 \end{cases} \Rightarrow (a,b) = \underline{\left(\frac{1}{2}, -1\right)}$.

3. Since the vertices of the ellipse are y-intercepts, we let $x = 0$, resulting in
$8y^2 - 84y + 108 = 0 \Rightarrow 2y^2 - 21y + 27 = (2y-3)(y-9) = 0$ Thus, the vertices are $\underline{\mathbf{(0,9), \left(0, \frac{3}{2}\right)}}$.
This equation could have been solved by *reducing the roots by a factor*, in this case 3.
$2y^2 - (21/3)y + (27/3^2) \Rightarrow 2y^2 - 7y + 3 = (2y-1)(y-3) = 0$ with roots of ½ and 3. So the actual roots are 3/2 and 9. [21 was twice the sum of the roots and 27 twice the product of the roots.]
Alternatively, this is a <u>vertical</u> ellipse $\left(\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1\right)$, so the major axis is a vertical segment. We need to find the center and the value of a.
Completing the square, $9x^2 + 8y^2 - 84y + 108 = 0$

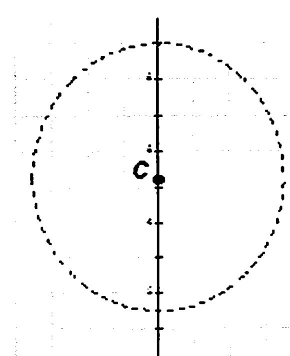
$$\Leftrightarrow 9x^2 + 8\left(y^2 - \frac{21}{2}y + \frac{21^2}{4^2}\right) = -108 + 8\left(\frac{21^2}{4^2}\right)$$

$$\Leftrightarrow 9x^2 + 8\left(y - \frac{21}{4}\right)^2 = -108 + 8\left(\frac{21^2}{4^2}\right) = \frac{-4^3(27) + 8(21)^2}{4^2}$$

$$\Leftrightarrow \frac{2^3 3^2(-24+49)}{2^4} = \frac{25(3^2)}{2}$$

Thus, the center (h, k) is $\left(0, \frac{21}{4}\right)$ and

$a^2 = \frac{9(25)}{16} \Rightarrow a = \frac{15}{4}$ and the endpoints are $\underline{\mathbf{(0,9), \left(0, \frac{3}{2}\right)}}$.



As expected with the numerical values of the coefficients of x^2 and y^2 so close together, the ellipse very closely resembles a circle.

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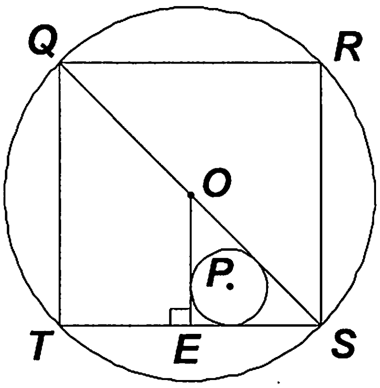
TEAM ROUND



1. A diagonal of square $QRST$ is a diameter of circle O. Without loss of generality (WLOG), let the side of square $QRST$ be 2. Then: $SE = OE = 1$ and $OS = \sqrt{2}$

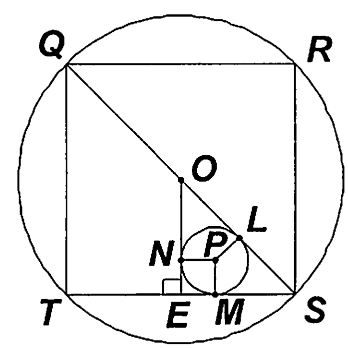
   $A(circle\ O) = \pi\left(\sqrt{2}\right)^2 = 2\pi$

   Appealing to the formula $Area(\Delta) = r \cdot s$, where r denotes the radius of the inscribed circle and s denotes the **semi**-perimeter of the triangle,

   for ΔOES, $\frac{1}{2}\cdot 1\cdot 1 = r\left(\frac{2+\sqrt{2}}{2}\right) \Rightarrow r = \frac{1}{2+\sqrt{2}}$ and

   $B = \text{Area}(\text{circle } P) = \pi\left(\frac{1}{2+\sqrt{2}}\right)^2 = \frac{\pi}{2\left(3+2\sqrt{2}\right)}$

   Equating, cancelling the common factor of π and cross multiplying,
   $A = kB \Leftrightarrow k = \underline{\mathbf{4\left(3+2\sqrt{2}\right)}}$ or equivalent.



   Alternatively, let the radius of the small circle be $PL = PM = PN = 1$.
   Note L is the midpoint of $\overline{OS}$ and, since tangents to a circle from any external point are congruent, $OL = ON$ and $SL = SM$, we let x denote the length of all 4 of these segments.
   Applying the Pythagorean Theorem to ΔOES,

   $2(x+1)^2 = (2x)^2 \Rightarrow 2x^2 + 4x + 2 = 4x^2$

   $\Rightarrow x^2 - 2x - 1 = 0$

   $\Rightarrow x = \frac{2\pm\sqrt{8}}{2} \Rightarrow x = 1+\sqrt{2}$.

   Since the area of circle P is π, $A = kB \Leftrightarrow \pi(2x)^2 = k\pi \Rightarrow k = 4x^2 = 4\left(1+\sqrt{2}\right)^2 = \underline{\mathbf{4\left(3+2\sqrt{2}\right)}}$.

Detailed Solutions for GBML Meet 4 - JANUARY 2016

TEAM ROUND

2. $\dfrac{x+2}{(x+1)(x-3)}+\dfrac{x-1}{(x-2)(x-3)}+\dfrac{x+3}{(x+1)(x-2)}+\dfrac{x^2(x-4)+(x+8)}{(x+1)(x-2)(x-3)}\geq 0$

$\Leftrightarrow \dfrac{(x+2)(x-2)+(x-1)(x+1)+(x-3)(x+3)+x^2(x-4)+(x+8)}{(x+1)(x-2)(x-3)}\geq 0$

$\Leftrightarrow \dfrac{3x^2-14+x^3-4x^2+x+8}{(x+1)(x-2)(x-3)}\geq 0 \Leftrightarrow \dfrac{x^3-x^2+x-6}{(x+1)(x-2)(x-3)}\geq 0$

$\Leftrightarrow \dfrac{(x-2)(x^2+x+3)}{(x+1)(x-2)(x-3)}\geq 0 \Leftrightarrow \dfrac{(x-2)\left(\left(x+\frac{1}{2}\right)^2+\frac{11}{4}\right)}{(x+1)(x-2)(x-3)}\geq 0$ Clearly, $x=-1,2,-3$ must be avoided and, for all x, $\left(x+\dfrac{1}{2}\right)^2+\dfrac{11}{4}>0$ and can be ignored. Thus, the original inequality is equivalent to $\dfrac{1}{(x+1)(x-3)}\geq 0$, provided $x\neq 2$. The solution to this simplified inequality is $-1<x<3$. However, this interval contains $x=2$ which must be excluded.

Possible answers: $\underline{\mathbf{-1<x<3 \text{ and } x\neq 2}}$, $\underline{\mathbf{-1<x<2,\ 2<x<3}}$, $\underline{\mathbf{(-1,2),(2,3)}}$

The "and" may not be replaced by "or". The commas may be replaced by "or".
Commas replaced by "and" should also be accepted since the assertion is that *both intervals contain x-values which satisfy the given equation.****

*** A **solution set** is the union of two disjoint intervals. The union of two disjoint intervals is always expressed with "or". Any solution is either in one interval or it is in the other interval. No solution is in both. Since a solution set was not requested, the "and" interpretation is reasonable.

3. $t_7-t_3=4d=(5y+6)-(3y-4)=2y+10 \Rightarrow y=2d-5$

$t_6-t_5=(x+z+3y)-(7x-6)=d \Leftrightarrow -6x+z+3y+6=d$

$\Leftrightarrow -2d-18+z+6d-15+6=d \Rightarrow z=27-3d$

Thus, $(t_3,t_4,t_5)=(3y-4,4x+3,7x-6)=\left(6d-19,\dfrac{4d+45}{3},\dfrac{7d+45}{3}\right)$.

Since $d=t_4-t_3=t_5-t_4$ we have $\dfrac{4d+45}{3}-(6d-19)=\dfrac{7d+45}{3}-\dfrac{4d+45}{3}=\dfrac{3d}{3}=d$

$\Rightarrow 4d+45-18d+57=3d \Rightarrow 17d=102 \Rightarrow d=6 \Rightarrow (x,y,z)=(5,7,9)$

$\Rightarrow (t_1,t_2,t_6,t_7)=\underline{\mathbf{(5,11,35,41)}}$.