GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2019

ROUND 1-Volume and Surface Area of Solids

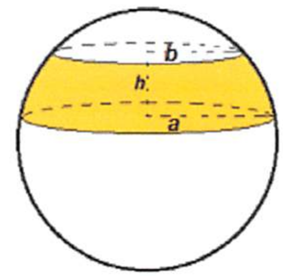
1. ______________________ units^2

2. ______________________ units^3

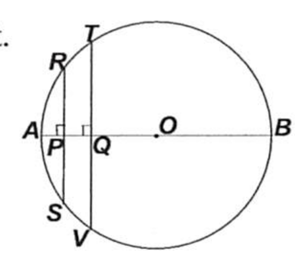
3. ______________________

1. A right circular cone has a base with area of $576\pi\ \text{units}^2$ and a slant height of 40 units. Compute the area of a cross-section 12 units from the base.

2. Consider the trisection points of the 12 edges of a cube whose edges have length 9 units. Each vertex and the three trisection points closest to that vertex determine a pyramid with an equilateral triangular base. Compute the volume of the solid with 14 faces formed by removing the pyramids at each vertex of the cube.



3. A *spherical segment* is a region in the interior of the sphere trapped between two parallel planes. Its volume can be computed using the formula $V=\dfrac{\pi h}{6}\left(3a^2+3b^2+h^2\right)$.
In circle O, chords $\overline{RS}$ and $\overline{TV}$, which are perpendicular to diameter $\overline{AB}$, are rotated about $\overline{AB}$ to generate a spherical segment. $\overline{AP}$, $\overline{PQ}$, $\overline{QO}$, $\overline{RS}$, and $\overline{TV}$ have integer lengths and $AP:PQ:QO=1:1:3$. The volume of this spherical segment is $k\pi\ \text{units}^3$, where k is an integer. Compute the <u>minimum</u> value of k.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2019

ROUND 2-Inequalities and Absolute Value

1. ______________________________

2. (________ , ________ , ________)

3. ______________________________

1. Given: $\begin{cases} A = \{x : |x-1| < 20\} \\ B = \{x : |x+15| \geq 19\} \end{cases}$

   Compute the <u>maximum</u> distance between points with integer coordinates in the intersection of sets A and B.

2. The set of points in the graph below satisfies an inequality of the form $a < |x-b| \leq c$, where a, b, and c are integers. Compute the ordered triple (a,b,c).

3. Solve for x over the reals. $\dfrac{2x-1}{x} + 1 \leq \dfrac{1}{2x+1}$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2019

ROUND 3 - Similar Polygons, Circles and Area

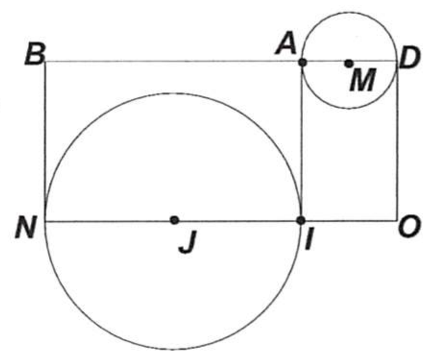
1. ______________________

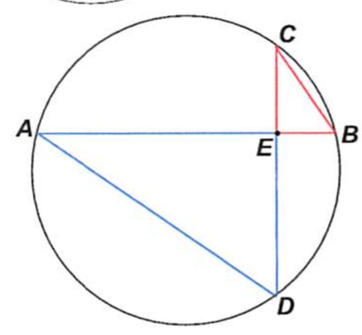
2. ______________________

3. ______________________

DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE.

1. $\overline{AI}$ divides a 9 x 30 rectangle $BDON$ into two similar, but not congruent, rectangles. Circles M and J have diameters $\overline{AD}$ and $\overline{IN}$, respectively. The sum of the areas of circle M and circle J is $k\pi$. Compute k.





2. Given: $AB = 28,\ CD = 22,\ \dfrac{BE}{CE} = \dfrac{2}{3},$ $\overline{AB} \cap \overline{CD} = \{E\},\ \overline{CD} \perp \overline{AB}$

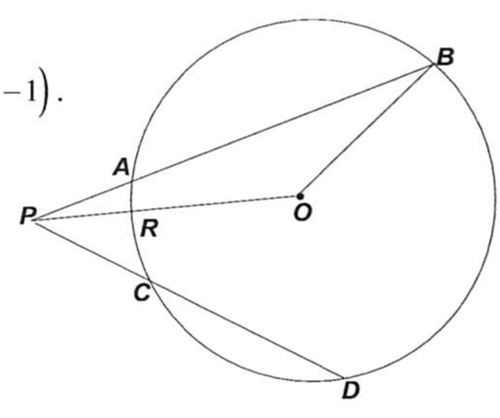
   Compute the positive difference between the areas of ΔBCE and ΔADE.

3. In circle O, $PA = 3$, $CD = 5$, $PC = 4$, and $PR = 6\left(\sqrt{2} - 1\right)$.

   The following are collinear sets of points:

   $\{P, A, B\}, \{P, R, O\}, \{P, C, D\}$

   Compute the area of ΔPOB.



GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2019

ROUND 4 - Sequences and Complex Numbers

1. ______________________________

2. ______________________________

3. ______________________________

1. Given: $z = a + bi$
   Compute <u>all</u> possible values of $a + b$, if $z + \bar{z} = z^{-1}$.

2. 2019 is the middle term in an arithmetic sequence of 2019 terms with a common difference of $-\frac{8}{3}$. Compute the <u>largest</u> integer in this sequence.

3. If $\begin{cases} T_n = T_{n-1} \cdot i + 1 \\ T_0 = 1 \end{cases}$, compute $\sum_{n=2019}^{n=2025} T_n$

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2019

ROUND 5-Conics

1. (________ , ________ , ________)

2. ______________________________

3. ______________________________

1. Let a, b, and c denote the shortest distance froma point on the graph of $x^2 + y^2 - 8x - 6y + 21 = 0$ to the x-axis, y-axis, and origin, respectively. Compute the ordered triple (a, b, c).

2. Given lines $L_1 : \left\{(x, y) \mid \frac{4}{3}x - \frac{5}{2}y + k = 0\right\}$ and $L_2 : \left\{(x, y) \mid \frac{8}{3}x - 5y + k = 0\right\}$
Compute all values of k so that the distance from the y-intercept of L_2 to L_1 is $\frac{18}{17}$ units.

3. Compute $n > 0$, for which $x = n$ defines one of the directrices of the ellipse whose equation is $\frac{(x-1)^2}{25} + \frac{(y+2)^2}{9} = 1$.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2019

TEAM ROUND

3 pts. 1. ______________________

3 pts. 2. ______________________

4 pts. 3. ______________________

1. The lengths of the edges of three cubes are three consecutive odd integers. The cubes are resting on a table, centeredone on top of the other; the smallest on the top, the largest on the bottom. Excluding the bottom face of the largest cube, the total exposed surface area is 1125 square units. Compute the volume of the middle cube.

2. Compute the area of the triangle whose vertices are the focus and x-intercepts of the parabola whose equation is $y=(2x-1)(2x+5)$.

3. The following sequence of literal expressions form an arithmetic progression of 8 terms.
$2y,\ \frac{6x-5}{2},\ 4z-3,\ 12y+9,\ 8x-1,\ 6w-8,\ 12x-3,\ 10z+4$
Compute the ordered quintuple (w,x,y,z,t_{19}), where t_{19} denotes the 19^{th} term in the progression.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 4 – JANUARY 2019

Answer Sheet

Round 1

1. 225π
2. 693
3. 342

Round 2

1. 16
2. $(7,-2,14)$
3. $-\frac{1}{2}<x\le-\frac{\sqrt{6}}{6}$ or $0<x\le\frac{\sqrt{6}}{6}$

   Alternately, $\left(-\frac{1}{2},-\frac{\sqrt{6}}{6}\right]$ or $\left(0,\frac{\sqrt{6}}{6}\right]$

Round 3

1. $\frac{369}{2}$ or 184.5
2. 180
3. $9\sqrt{7}$

Round 4

1. $\pm\frac{\sqrt{2}}{2}$
2. 4707
3. $4+3i$

Round 5

1. $(1,2,3)$
2. ± 6
3. $\frac{29}{4}$ ($7\frac{1}{4}$ or 7.25)

Team Round

1. 729 (3 pts)
2. $\frac{429}{32}$ (3 pts)
3. $(8.5, 4.5, 1.5, 5.5, 147)$ (4 pts)

Detailed Solutions for GBML Meet 4 - JANUARY 2019

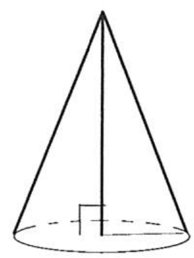
ROUND 1

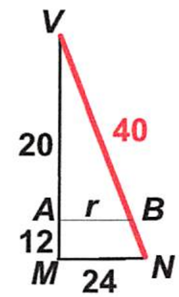
1. $\pi r^2 = 576\pi \Rightarrow r = 24$

   Using the Pythagorean Theorem or common Pythagorean Triples, the height of the cone is 32.

   $h = \sqrt{40^2 - 24^2} = \sqrt{1600 - 576} = \sqrt{1024} = \sqrt{2^{10}} = 2^5 = 32$,

   Or $(\_\_,\ 24,\ 40) = 8(\_,3,5) \Rightarrow h = 8(4) = 32$.



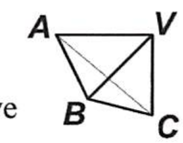


   Using similar triangles,

   $$\Delta VAB \sim \Delta VMN \Rightarrow \frac{20}{32} = \frac{r}{24} \Rightarrow r = \frac{20 \cdot 24}{4 \cdot 8} = 5 \cdot 3 = 15.$$

   Thus, the area of the cross-section is $\underline{\mathbf{225\pi}}$.

2. Let's examine one of the pyramids which is to be removed. Any face of this pyramid can be considered a base. However, using ΔABCas the base would be inconvenient, since we would then have to determine the distance from V to ΔABC.Since the edges of the cube intersecting at V form three right angles, choosing ΔVBC as the base makes $\overline{AV}$ the height! Thus, the volume of the pyramid is $\frac{1}{3} \cdot 3\left(\frac{1}{2} \cdot 3^2\right) = 4.5$, and the volume of the required region is $9^3 - 8(4.5) = 729 - 36 = \underline{\mathbf{693}}$.



3. Given: $AP : PQ : QO = 1:1:3$ and all integer lengths

   $(AP, PQ, QO) = (x, x, 3x) \Rightarrow OB = 5x$

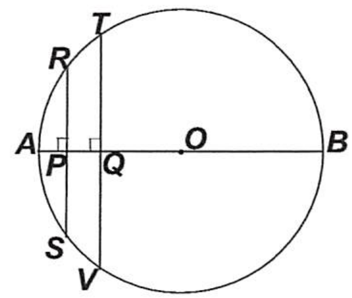
   By the product-chord theorem,

   $AP \cdot PB = RP \cdot PS = RP^2 = x(9x) = 9x^2$

   $AQ \cdot QB = TQ \cdot QV = TQ^2 = 2x(8x) = 16x^2$

   $$V = \frac{\pi h}{6}\left(3a^2 + 3b^2 + h^2\right) = \frac{x}{6}\left(27x^2 + 48x^2 + x^2\right)\pi = \left(\frac{x}{6} \cdot 76x^2\right)\pi \Rightarrow k = \frac{38}{3}x^3.$$

   Clearly, the <u>minimum</u> value of x for which k is an integer is 3, which gives us $k = \underline{\mathbf{342}}$.

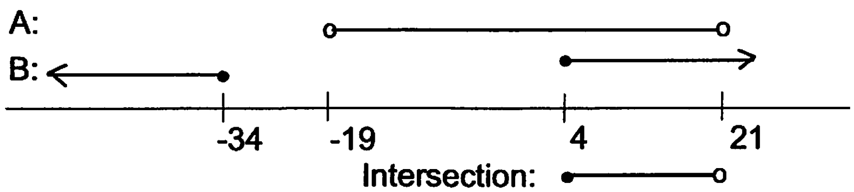


Detailed Solutions for GBML Meet 4 - JANUARY 2019

ROUND 2

1. $A=\{x:|x-1|<20\} \Leftrightarrow -20<x-1<20 \Leftrightarrow \boxed{-19<x<21}$

$B=\{x:|x+15|\geq 19\} \Leftrightarrow x+15\leq -19 \text{ or } x+15\geq 19 \Leftrightarrow \boxed{x\leq -34 \text{ or } x\geq 4}$



Thus, $A\cap B=\{x:4\leq x<21\} \Rightarrow$ maximum distance $=20-4=$ **16**.

2. The graph 

shows points which are symmetric about $x=-2$. Specifically, greater than 7 units from -2, but at most 14 units from -2. The first condition is equivalent to $|x+2|>7$. The second condition is equivalent to $|x+2|\leq 14$. These may be combined as $7<|x+2|\leq 14$, which gives us $(a,b,c)=$ **(7, −2, 14)**.

3. $\dfrac{2x-1}{x}+1\leq\dfrac{1}{2x+1} \Leftrightarrow \dfrac{2x-1}{x}-\dfrac{1}{2x+1}+1\leq 0 \Leftrightarrow \dfrac{(2x-1)(2x+1)-x+x(2x+1)}{x(2x+1)}\leq 0$

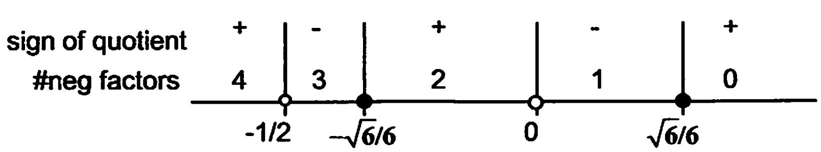
Simplifying the numerator, we have $\dfrac{4x^2-1-x+2x^2+x}{x(2x+1)}\leq 0 \Leftrightarrow \dfrac{6x^2-1}{x(2x+1)}\leq 0$

$\Leftrightarrow \dfrac{(\sqrt{6}x+1)(\sqrt{6}x-1)}{x(2x+1)}\leq 0$. The critical values are $\pm\dfrac{1}{\sqrt{6}}=\pm\dfrac{\sqrt{6}}{6}$, 0, $-\dfrac{1}{2}$.

$2.5^2=6.25 \Rightarrow \sqrt{6}<2.5 \Rightarrow \dfrac{\sqrt{6}}{6}<\dfrac{2.5}{6}<.5$ and $-\dfrac{\sqrt{6}}{6}>\dfrac{2.5}{6}>-.5$.

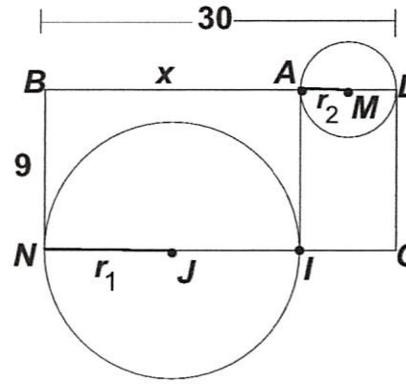
Thus, on the number line, the correct order is $-\dfrac{1}{2}$, $-\dfrac{\sqrt{6}}{6}$, 0, $\dfrac{\sqrt{6}}{6}$.

At the extreme left, all 4 factors are negative; at the extreme right, all 4 factors are positive. Moving along the number line from left to right, as each critical value is passed, one less term in the quotient is negative. Since the number of negative factors determines the sign of the factored quotient, the following diagram gives us the intervals where the quotient is negative or zero. $\underline{-\dfrac{1}{2}<x\leq -\dfrac{\sqrt{6}}{6} \text{ or } 0<x\leq\dfrac{\sqrt{6}}{6}}$ Also: $\underline{\left(-\dfrac{1}{2},-\dfrac{\sqrt{6}}{6}\right] \text{ or } \left(0,\dfrac{\sqrt{6}}{6}\right]}$



Detailed Solutions for GBML Meet 4 - JANUARY 2019

ROUND 3



1. Since the required sum is $\pi\left(r_1^2+r_2^2\right)$, without loss of generality, we assume $r_1 > r_2$.

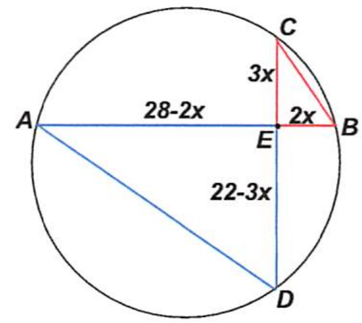
$BAIN \sim DOIA \Rightarrow \dfrac{BA}{DO}=\dfrac{AI}{OI} \Rightarrow \dfrac{x}{9}=\dfrac{9}{30-x}$

$\Rightarrow x^2-30x+81=(x-27)(x-3)=0 \Rightarrow x=27$

Thus,

$\pi\left(r_1^2+r_2^2\right)=\pi\left(\left(\dfrac{27}{2}\right)^2+\left(\dfrac{3}{2}\right)^2\right)=\dfrac{738}{4}\pi \Rightarrow k=\underline{\mathbf{\dfrac{369}{2}}}$ or $\underline{\mathbf{184.5}}$.

2. By the product-chord theorem,
$$\begin{aligned}&2x(28-2x)=3x(22-3x)\\ &\Leftrightarrow 56x-4x^2=66x-9x^2\\ &\Leftrightarrow 5x^2-10x=5x(x-2)=0\\ &\Rightarrow x=2\end{aligned}.$$



Thus, the areas of the two triangles are $\dfrac{1}{2}\cdot 6\cdot 4$ and $\dfrac{1}{2}\cdot 24\cdot 16$, which produces a difference of $\dfrac{1}{2}\cdot 4\cdot 6(4\cdot 4-1)=12\cdot 15=\underline{\mathbf{180}}$.

3. Let $\overrightarrow{PO}$ intersect circle O in point S and $AB=x$.
By the secant-chord theorem,
$PA(PB)=PR(PS)=PC(PD)$. Thus,

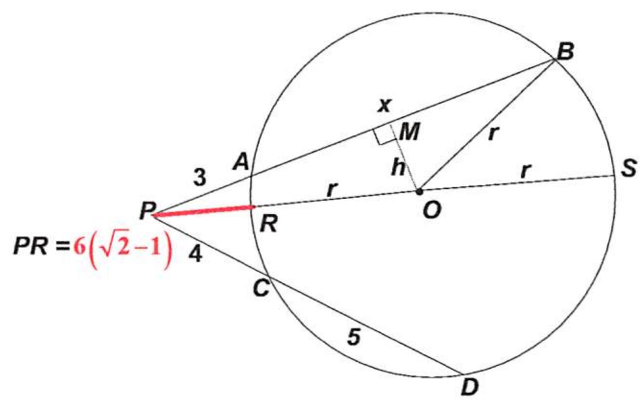
$4\cdot 9=3(x+3)=6\left(\sqrt{2}-1\right)\left(6\left(\sqrt{2}-1\right)+2r\right)$

$\Rightarrow x=9$

$\Rightarrow 6=\left(\sqrt{2}-1\right)\left(6\left(\sqrt{2}-1\right)+2r\right)$

$\Rightarrow 3=\left(\sqrt{2}-1\right)\left(3\left(\sqrt{2}-1\right)+r\right)$

$\Rightarrow r=\dfrac{3}{\sqrt{2}-1}-3\sqrt{2}+3=3\left(\sqrt{2}+1\right)-3\sqrt{2}+3=6$.



The required area is $\dfrac{1}{2}\cdot 12\cdot h=6h$.

In ΔMOB, $h^2=6^2-4.5^2=36-20.25=15.75=\dfrac{63}{4} \Rightarrow h=\dfrac{3\sqrt{7}}{2} \Rightarrow \underline{\mathbf{9\sqrt{7}}}$.

Detailed Solutions for GBML Meet 4 - JANUARY 2019

ROUND 4

1. $z+\bar{z}=z^{-1} \Leftrightarrow (a+bi)+(a-bi)=\dfrac{1}{a+bi} \Leftrightarrow 2a=\dfrac{a-bi}{a^2+b^2} \Leftrightarrow 2a+0i=\dfrac{a}{a^2+b^2}-\dfrac{b}{a^2+b^2}i$

   Equating the imaginary components, $0=\dfrac{-b}{a^2+b^2} \Rightarrow b=0$

   Equating the real components, $2a=\dfrac{a}{a^2+b^2}=\dfrac{a}{a^2}=\dfrac{1}{a} \Rightarrow 2a^2=1 \Rightarrow a=\pm\dfrac{\sqrt{2}}{2}$.

   Thus, $a+b=$ $\underline{\pm\dfrac{\sqrt{2}}{2}}$.

   Alternately (and more simply), $z+\bar{z}=z^{-1} \Leftrightarrow 2a=\dfrac{1}{a+bi}$.

   Now multiplying both sides by $a+bi$, we have $2a^2+2abi=1=1+0i \Rightarrow \begin{cases} 2a^2=1 \\ 2ab=0 \end{cases}$

   $\Rightarrow b=0,\ a=\pm\dfrac{\sqrt{2}}{2}$, and the same result follows.

2. There will be 1009 terms both before and following the middle term t_{1010}. Since $3\left(-\dfrac{8}{3}\right)$ is an integer, every third term will be an integer. Thus, $t_{1007}, t_{1004}, t_{1001}, \ldots, t_2$ are integers.
   The subscripts are of the form $1010-3k$ and $1010-3(336)=2$.
   The largest integer value is $t_2=2019+8(336)=2019+2688=\underline{\mathbf{4707}}$.

3. $\begin{cases} T_n=T_{n-1}\cdot i+1 \\ T_0=1 \end{cases} \Rightarrow T_1=1+i,\ T_2=i,\ T_3=0,\ T_4=1$

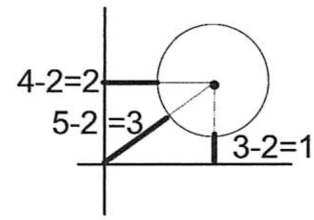
   This recursive definition generates a cycle of 4 distinct values which sum to $2+2i$, regardless of which value we start with. The given summation requires that we add 7 consecutive values. We need determine only which 7.

   $T_0=T_4=T_8=\ldots=T_{2020}=1$

   $\displaystyle\sum_{n=2019}^{n=2025} T_n=T_{2019}+\sum_{n=2020}^{n=2023} T_n+T_{2024}+T_{2025}=0+(2+2i)+1+(1+i)=\underline{\mathbf{4+3i}}$.

Detailed Solutions for GBML Meet 4 - JANUARY 2019

ROUND 5



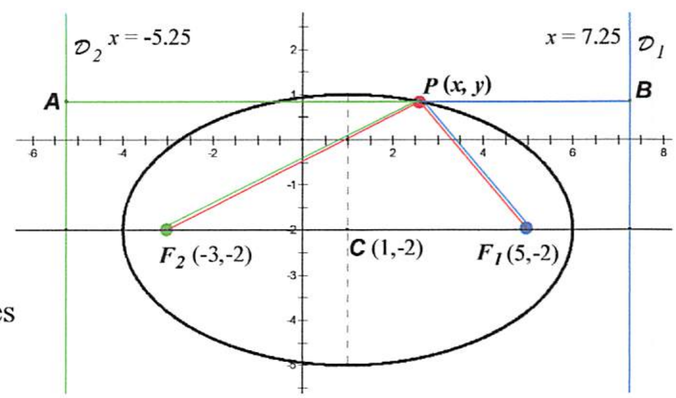
1. $x^2+y^2-8x-6y+21=0 \Leftrightarrow (x-4)^2+(y-3)^2=4$
   This is the equation of a circle with center at (4, 3) and radius 2.
   Thus, $(a,b,c)=\underline{\mathbf{(1,2,3)}}$.

2. Eliminating the fractional coefficients, $\begin{matrix} L_1: 8x-15y+6k=0 \\ L_2: 8x-15y+3k=0 \end{matrix}$

   The y-intercept of L_2 is at $\left(0,\frac{k}{5}\right)$. Using the point-to-line distance formula,

   $$\frac{\left|8\cdot 0-15\cdot\frac{k}{5}+6k\right|}{\sqrt{8^2+15^2}}=\frac{18}{17}\Rightarrow\frac{|3k|}{17}=\frac{18}{17}\Rightarrow|k|=6\Rightarrow k=\underline{\pm 6}.$$

3. $\frac{(x-1)^2}{25}+\frac{(y+2)^2}{9}=1$ defines a horizontal ellipse with center at $(1,-2)$, major axis of length 10 ($a=5$) and minor axis of length 6 ($b=3$). Since, for all ellipses, $a^2=b^2+c^2$, we have $c=4$. By definition, eccentricity $e=\frac{c}{a}=\frac{4}{5}$ and the distance from the center to either directrix is $\frac{a}{e}=\frac{5}{4/5}=\frac{25}{4}$, implying the equation of the directrix is $x=1+\frac{25}{4}=\underline{\mathbf{\frac{29}{4}}}$ (or $\underline{\mathbf{7\frac{1}{4}}}=\underline{\mathbf{7.25}}$). The ellipse is commonly defined in two different ways:



   (1) The set of points the sum of whose distances from two fixed points (foci) is equal to a constant ($2a$).

   $\sqrt{(x+3)^2+(y+2)^2}+\sqrt{(x-5)^2+(y+2)^2}=10$ is equivalent to the given equation.

   (2) Focus-Directrix-Eccentricity:The set of points for which the ratio of the distance from a fixed point (a focus) to the distance from a fixed line (a directrix) equals a constant (the eccentricity – defined as $e=\frac{c}{a}$).

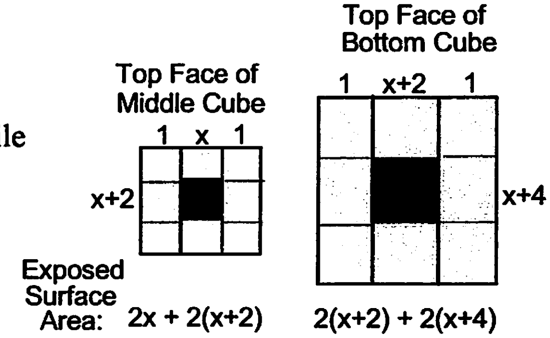
$n=\frac{29}{4}$: $\sqrt{(x-5)^2+(y+2)^2}=\frac{4}{5}\cdot\left|\frac{29}{4}-x\right|$ or $n=-\frac{21}{4}$: $\sqrt{(x+3)^2+(y+2)^2}=\frac{4}{5}\cdot\left|x+\frac{21}{4}\right|$

You should verify that each of these equations is equivalent to the given equation.

Detailed Solutions for GBML Meet 4 - JANUARY 2019

TEAM ROUND

1. Let x denote the length of an edge of the smallest cube. The top cube has 5 lateral faces exposed, while the bottom two cubes have only 4 lateral faces exposed. The top face of the $(x+2)$ by $(x+2)$ middle cube and the top face of the $(x+4)$ by $(x+4)$ bottom cube are shown in the diagram at the right.



$$5(x-2)^2+4(x)^2+4(x+2)^2+2x+4(x+2)+2(x+4)=1125$$
$$13x^2+56x+96=1125$$
$$13x^2+56x-1029=0 \Leftrightarrow (13x+147)(x-7)=0$$
$$x=7 \Rightarrow 9^3=\underline{\mathbf{729}}$$

Alternate Solution #1:Let $x-2$ denote the length of an edge of the smallest cube. Then:
$$5(x-2)^2+4(x)^2+4(x+2)^2+\left(x^2-(x-2)^2\right)+\left((x+2)^2-x^2\right)=1125$$
Expanding, $5\left(x^2-4x+4\right)+4x^2+4\left(x^2+4x+4\right)+(4x-4)+(4x+4)=1125$

Combining like terms, $13x^2+4x-1089=0$

Since $1089=9\cdot 121$ and one solution must be on odd integer, we have
$\left(13x+\boxed{121}\right)\left(x-\boxed{9}\right)=0$.

Thus, the volume of the middle cube is $9^3=\underline{\mathbf{729}}$.

Alternate Solution #2: Let A, B, and C denote the sides of the 3 cubes, where $A<B<C$. Then: The exposed surface area will be
$$5A^2+4\left(B^2+C^2\right)+\left(B^2-A^2\right)+\left(C^2-B^2\right)=4A^2+4B^2+5C^2=1125.$$
Clearly, the exposed surface area of cubes with edges of 1, 3, 5 would be too small. To avoid tedious arithmetic, examine the **units digit** of $4A^2+4B^2+5C^2$ for consecutive odd integers. If the units digit is not 5, move on; if it is 5, check the arithmetic.

	$4A^2$	$4B^2$	$5C^2$	**Units Digit**	**Verdict**
3, 5, 7	6	0	5	1	rejected
5, 7, 9	0	6	5	1	rejected
7, $\boxed{9}$, 11	6	4	5	5	Possible

Testing, $4(49+81)+5(121)=520+605=1125$ Bingo! $\Rightarrow 9^3=\underline{\mathbf{729}}$.

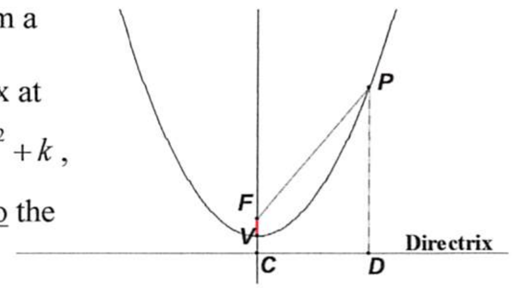
Detailed Solutions for GBML Meet 4 - JANUARY 2019

TEAM ROUND - continued

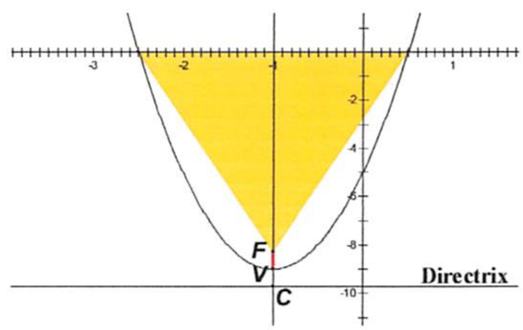
2. The x-intercepts are clearly at $\left(\frac{1}{2},0\right)$ and $\left(-\frac{5}{2},0\right)$.

$$y=(2x-1)(2x+5) \Leftrightarrow 4x^2+8x-5=4\left(x^2+2x+1\right)-5-4=4(x+1)^2-9$$

By definition, a parabola is a set of points equidistant from a fixed point and a fixed line.
For a parabola with a vertical axis of symmetry and vertex at (h, k), the standard form of the equation is $y=\frac{1}{4a}(x-h)^2+k$,
where a denotes the *directed* distance from the vertex V to the focus F.
If F is above V, then $a>0$ and the parabola opens UP.
If F is below V, then $a<0$ and the parabola opens DOWN.



We have $(h,k)=(-1,-9)$ and $\frac{1}{4a}=4 \Rightarrow a=\frac{1}{16}$.
The focus F is at $\left(-1,-9+\frac{1}{16}\right)=\left(-1,-\frac{143}{16}\right)$.
[Since $VF=VC$, the equation of the directrix is $y=-9-\frac{1}{16}=-\frac{145}{16}$.]



The area of the required triangle is $\frac{1}{2}bh=\frac{1}{2}\cdot\left(\frac{1}{2}-\left(-\frac{5}{2}\right)\right)\cdot\frac{143}{16}=\frac{3\cdot 143}{2\cdot 16}=\mathbf{\frac{429}{32}}$.

It is left to you to verify that the equation of a parabola with a horizontal axis of symmetry is $x=\frac{1}{4a}(y-k)^2+h$, where the parabola opens to the right for $a>0$; and, opens to the left for $a<0$.

Detailed Solutions for GBML Meet 4 - JANUARY 2019

TEAM ROUND - continued

3. Given: $2y,\ \dfrac{6x-5}{2},\ 4z-3,\ 12y+9,\ 8x-1,\ 6w-8,\ 12x-3,\ 10z+4$

Since the second, fifth and seventh terms are all *x*-expressions, we can set up a system of two equations in terms of *x* and *d*, the common difference.

$$\begin{cases}(8x-1)-\left(\dfrac{6x-5}{2}\right)=3d\\(12x-3)-(8x-1)=2d\end{cases}\Leftrightarrow\begin{cases}16x-2-6x+5=6d\\4x-2=2d\end{cases}\Leftrightarrow\begin{cases}10x-6d+3=0\\d=2x-1\end{cases}$$

Substituting for *d*, $10x-6(2x-1)+3=0\Leftrightarrow -2x+9=0\Leftrightarrow x=4.5, d=8$

Now the arithmetic sequence is $\_\_, 11, \_\_, \_\_, 35, \_\_, 51, \_\_, \ldots$.

$d=8\Rightarrow 3,11,19,27,35,43,51,59,\ldots$. The 19th term is $3+18\cdot 8=147$.

$$\begin{cases}2y=3\\4z-3=19\\6w-8=43\end{cases}\Rightarrow (w,x,y,z,t_{19})=\underline{\mathbf{(8.5, 4.5, 1.5, 5.5, 147)}}$$

You should verify that $t_4=12y+9$ and $t_8=10z+4$ are correct.

> If $t_2=2x+2$, instead of $\dfrac{6x-5}{2}$, then $(x,d)=(4.5,8)$ is a solution, but so is (5, 9), which produces 3, 12, 21, 30, 39, 48, 57, 66, ... and a different quintuple.
> Are there other solutions as well? Ans: Yes, any ordered pairs of the form $(x,2x-1)$.
> Curious! Can you explain why this happened?

In honor of Martin J. Badoian, longtime author of Greater Boston Mathematics League questions and coach of the Canton High School math team since 1961.Marty passed away at his home in Sharon, MA on 10/27/2018. This is one of the last competition questions he authored in the summer of 2017, just before the path of totality of a solar eclipse cut a path from Oregon to South Carolina.It was not used in the 2017-18 competition, so it is appropriate that this year's competition ends with his question. He frequently wrote questions while enjoying vacation time with his wife Linda and his family in Islesboro, Maine.