GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 - FEBRUARY 2017

ROUND 1 - Arithmetic: Open

1. ____________________

2. ____________________

3. ____________________

1. Set A has 120 more subsets than set B. How many elements are there in set A?

   Recall: Set A is a subset of set B ($A \subseteq B$) if (and only if) every element in A is also in B.

2. The sum of two rational numbers is $6\frac{1}{4}$ and the sum of their reciprocals is $-\frac{2}{3}$. Compute the larger of the two numbers.

3. If N is a 3-digit number whose units digit is 3 and which has at least 2 odd digits, compute the probability that N is divisible by 3.

   Note: Assume that the hundreds digit may not be zero.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 - FEBRUARY 2017

ROUND 2 - Algebra 1: Open

1. ______________________

2. ______________________

3. ______________________

1. Given: $a\sqrt{3-x}-b=a-b\sqrt{3-x}$, where a and b are <u>nonzero</u> constants.
   For a unique value of c, if $a+b=c$, this equation is satisfied for all values of x.
   If $a+b\neq c$, then the only solution is $x=d$.
   Determine the ordered pair (c,d).

2. A giant watermelon weighed 20 pounds and was 99% water. After sitting in the hot sun for several hours, a significant amount of water evaporated so that the watermelon was only 96% water. Compute the new weight of the watermelon.

3. The sum of three positive numbers is 28, and one is twice another. Two times the largest number is 8 more than the sum of the two smallest numbers. Compute <u>all</u> possible values of the smallest number.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 - FEBRUARY 2017

ROUND 3 - Geometry: Open

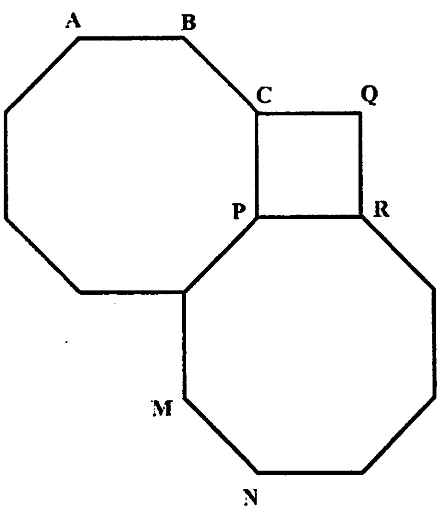
1. ______________________

2. ______________________

3. (__________ , __________)

1. Angle A has a measure of $(7x+2)°$ and angle B has a measure of $(9x-2)°$. The degree measure of the supplement of angle B is 20 degrees more than four times the degree measure of the complement of angle A. Compute the degree measure of the supplement of angle A.

2. Two regular octagons and a square have point P in common. If $PQ = 1$, compute the distance from point B to point M.



3. A circle is inscribed in a square, and then an equilateral triangle is inscribed in the circle. The perimeter of the square is 24 inches. The ratio of the area of the equilateral triangle to the area of the square is $A\sqrt{3} : B$, where A and B are relatively prime integers. Determine the ordered pair (A, B).

GREATER BOSTON MATHEMATICS LEAGUE

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ROUND 4 – Algebra 2: Open

1. ______________________

2. ______________________

3. ______________________

1. For all positive numbers a and b, function f satisfies $f(ab) = f(a) + f(b)$.
   If $f(2) = x$ and $f(5) = y$, compute the value of $f(1000)$ in terms of x and y.

2. Given the parabola $P: y^2 - 12x + 24 = 0$ and circle $C: x^2 + y^2 + 8x - 20 = 0$. Compute the length of a tangent drawn from the focus of the parabola P to the circle C.

3. Compute all ordered pairs of real numbers (x, y) satisfy the system $\begin{cases} x + xy + y = 11 \\ x^2 y + xy^2 = 30 \end{cases}$.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 – FEBRUARY 2017

ROUND 5 – Pre-Calculus: Open

1. ______________________

2. ______________________

3. ______________________

1. Compute, in radians, the values of x in the interval $-\pi < x < \pi$ for which

$$\left(2^{\sin^2 x}\right)\left(2^{\cos^2 x}\right)\left(2^{\tan^2 x}\right) = 2^4 .$$

2. Given: $\log_2 3 + \log_{36} 3 - \log_4 x = \log_{36} 18$
   Compute all real values of x for which this is true.

3. Given the following sequence: $6, x, y, 16, z$
   The first 3 elements of the sequence form an arithmetic sequence, while the middle 3 elements form a geometric sequence, and the final 3 form a harmonic sequence.
   Compute all possible real values of x, y, and z for which this is true, and write them as ordered triples (x, y, z).

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 - FEBRUARY 2017

TEAM ROUND

3 pts.	1.	______________
3 pts.	2.	______________
4 pts.	3.	______________

1. Determine N, the 99^{th} natural number which is divisible by 2 or 3, but not by 4.

2. In ΔABC, $m\angle A : m\angle B : m\angle C = 7:11:13$. ΔABC is inscribed in circle O.
   Tangent lines to circle O are drawn at points A, B, and C.
   These tangent lines intersect in points P, Q and R.
   In ΔPQR, $\angle P$ is the largest angle.
   Compute $m\angle PBC$ to the nearest degree.

3. Compute the numerical coefficient of x^{15} in the expansion of $\left(x^4+2x^2-3\right)^3\left(x^2-3x+2\right)^4$.

GREATER BOSTON MATHEMATICS LEAGUE

MEET 5 - FEBRUARY 2017

Answer Sheet

Round 1

1. 7
2. $\frac{15}{2}$
3. $\frac{4}{15}$

Round 2

1. $(0,2)$
2. 5
3. $6, \frac{16}{3}$ (Both answers are required.)

Round 3

1. 108°
2. $1+\sqrt{2}$
3. $(3,16)$

Round 4

1. $3x+3y$ or $3(x+y)$
2. $3\sqrt{5}$
3. $(5,1),(1,5),(2,3),(3,2)$

Round 5

1. $-\frac{2\pi}{3},-\frac{\pi}{3},\frac{\pi}{3},\frac{2\pi}{3}$
2. $\frac{9}{2}$
3. $(9,12,24), \left(1,-4,\frac{8}{3}\right)$

Team Round

1. 237 (3 pts)
2. 41 (3 pts)
3. −1476 (4 pts)

Detailed Solutions for GBML Meet 5 - FEBRUARY 2017

ROUND 1

1. Consider the sequence of powers of 2: $\boxed{1, 2, 4, 8, 16, 32, 64, 128, 256, \ldots}$
This sequence gives the number of subsets in sets containing 0, 1, 2, ..., 8, ... elements.
Background:
$\phi = \{\ \}$ = null set - the set with no elements is considered a subset of every set.
If n is the number of elements in a subset of a set S which has k elements, then $0 \le n \le k$.
By way of example, if T is a subset of set $S = \{a,b,c\}$, then $0 \le n \le 3$.

$n=0$: Pick none. [ϕ] $\binom{3}{0}=1$ $\qquad$ $n=1$: Pick 1 $[\ \{a\}, \{b\}, \{c\}\ ]$ $\binom{3}{1}=3$

$n=2$: Pick 2 $[\ \{a,b\}, \{a,c\}, \{b,c\}\ ]$ $\binom{3}{2}=3$ $\qquad$ $n=3$: Pick 3 $[\ \{a,b,c\}\ ]$ $\binom{3}{3}=1$

Total number of subsets = $1+3+3+1=8$. Remind you of Pascal's triangle?
We require 2 numbers in the boxed sequence above whose difference is 120.
Clearly, $128-8=2^7-2^3=120$ and set A has <u>7</u> elements.

2. Given: $\begin{cases} (1)\ a+b=\dfrac{25}{4} \\ (2)\ \dfrac{1}{a}+\dfrac{1}{b}=-\dfrac{2}{3} \end{cases}$

$$(2) \Rightarrow \frac{a+b}{ab}=\frac{25/4}{ab}=-\frac{2}{3} \Rightarrow ab=-\frac{75}{8} \Rightarrow a\left(\frac{25}{4}-a\right)=-\frac{75}{8}$$

$$\Rightarrow 8a^2-50a-75=(4a+5)(2a-15)=0 \Rightarrow a=-\frac{5}{4},\frac{15}{2}$$

Since the given system of equations are symmetric, i.e. a and b can be interchanged,
$(a,b)=\left(\frac{15}{2},-\frac{5}{4}\right)$ or vice versa. Thus, the larger number is $\underline{\mathbf{\frac{15}{2}}}$.

3. Let the 3-digit number be $\boxed{h}\boxed{t}\boxed{3}$.
Since divisibility by 3 requires that the sum of the digits be divisible by 3, $h+t$ must be a multiple of 3, namely, 3, 6, 9, 12, 15, or 18.

	3		6		9		12		15		18	
3,0	1	~~6,0~~		9,0	1	9,3	2	9,6	2	9,9	1	
2,1	2	5,1	2	8,1	2	~~8,4~~		8,7	2			
		~~4,2~~		7,2	2	7,5	2					
		3,3	1	6,3	2	~~6,6~~						
				5,4	2							
Totals:	3		3		9		4		4		1	⇒24

The chart at the right summarizes the possibilities:
Thus, the required probability is $\frac{24}{9\cdot 10}=\underline{\mathbf{\frac{4}{15}}}$.

Detailed Solutions for GBML Meet 5 - FEBRUARY 2017

ROUND 2

1. $a\sqrt{3-x}-b=a-b\sqrt{3-x} \Leftrightarrow a\sqrt{3-x}+b\sqrt{3-x}=a+b$

   $\Leftrightarrow (a+b)\left(\sqrt{3-x}\right)=a+b$

   If $a+b=0$, this is a trivial identity for all values of x.

   If $a+b\neq 0$, then $\sqrt{3-x}=1 \Rightarrow 3-x=1 \Rightarrow x=2$.

   Therefore, $(c,d)=\underline{(\mathbf{0,2})}$.

2. Let w and m denote the amount of water and the amount of melon, respectively. Then:

   $$\begin{cases} w+m=20 \\ \dfrac{w}{20}=\dfrac{99}{100} \end{cases} \Rightarrow 5w=99 \Rightarrow (w,m)=(19.8,.2)$$

   Suppose x lbs of water evaporated after the melon sat in the sun.

   $$\frac{19.8-x}{20-x}=\frac{96}{100} \Rightarrow 1980-100x=1920-96x \Rightarrow 4x=60 \Rightarrow x=15 \Rightarrow w_{new}=19.8-15=4.8 \text{ lbs.}$$

   Thus, the watermelon weighs $4.8+0.2=\underline{5}$ lbs.

3. Let the three numbers be $a, 2a$, and $28-3a$. Either $2a$ or $28-3a$ is the largest number.

   Case 1: ($2a$ is the largest)

   $2(2a)=8+(a+28-3a) \Leftrightarrow 4a=36-2a \Leftrightarrow a=6$

   $\Rightarrow$ 6,12,10 Smallest: **6**

   Check: $2(12)=8+(6+10)$

   Case 2: ($28-3a$ is the largest)

   $2(28-3a)=8+(a+2a) \Leftrightarrow 56-6a=3a+8 \Leftrightarrow a=\dfrac{16}{3}$

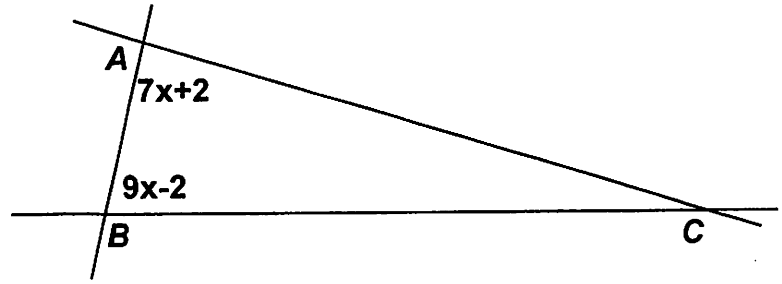
   $\Rightarrow \dfrac{16}{3},\dfrac{32}{3},12$ Smallest: $\underline{\mathbf{\dfrac{16}{3}}}$

   Check: $2(12)=8+\left(\dfrac{16}{3}+\dfrac{32}{3}\right)$

   Be sure to check out the addendum to this question!

Detailed Solutions for GBML Meet 5 - FEBRUARY 2017

ROUND 3



1. $180-(9x-2)=4(90-(7x+2))+20$

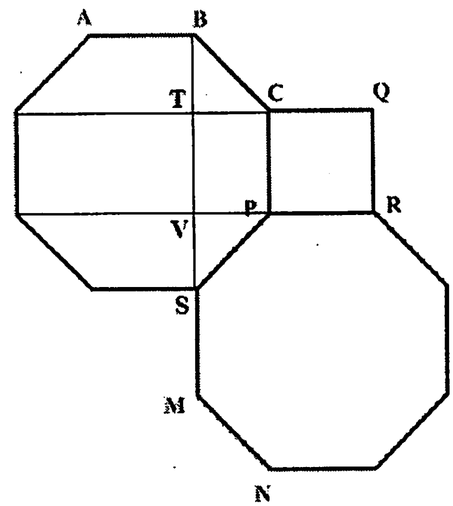
   $\Leftrightarrow 182-9x=4(88-7x)+20$

   $\Leftrightarrow 182-9x=372-28x$

   $\Leftrightarrow 19x=190$

   $\Leftrightarrow x=10$

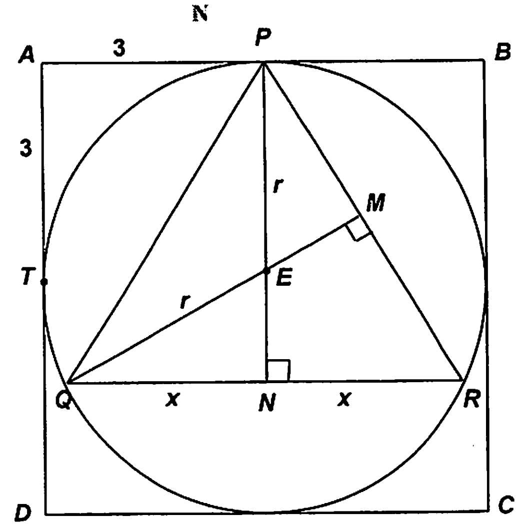
   $\Rightarrow m\angle A=7\cdot 10+2=72 \Rightarrow$ supplement = **<u>108°</u>**



2. $PQ=1 \Rightarrow PC=SM=\dfrac{\sqrt{2}}{2}$.

   $m\angle BCP=135° \Rightarrow \Delta BCT$ is an isosceles right triangle and $BT=VS=\dfrac{1}{2}$

   Thus, $BM=2\left(\dfrac{1}{2}+\dfrac{\sqrt{2}}{2}\right)=\underline{\mathbf{1+\sqrt{2}}}$.



3. Let E be the center of the circle and r denote the radius. Clearly, P is the midpoint of $\overline{AB}$ and $AP=AT=r=3$

   $\overline{PN}$ and $\overline{QM}$ are both altitudes and medians of equilateral ΔPQR and, therefore, $PE:EN=2:1 \Rightarrow EN=\dfrac{r}{2}=1.5$.

   In right ΔQEN,

   $3^2=\left(\dfrac{3}{2}\right)^2+x^2 \Rightarrow x^2=9-\dfrac{9}{4}=\dfrac{27}{4} \Rightarrow x=\dfrac{3}{2}\sqrt{3}$

   Thus, the area of ΔPQR is $\dfrac{1}{2}\cdot\dfrac{9}{2}\cdot 3\sqrt{3}=\dfrac{27}{4}\sqrt{3}$.

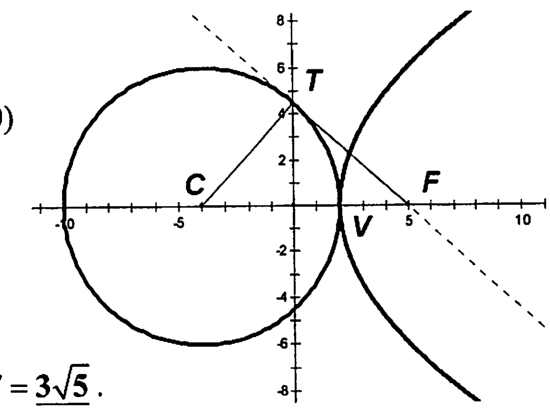
   $\dfrac{\frac{27}{4}\sqrt{3}}{36}=\dfrac{3\sqrt{3}}{16} \Rightarrow (A,B)=\underline{\mathbf{(3,16)}}$.

Detailed Solutions for GBML Meet 5 - FEBRUARY 2017

ROUND 4

1. $f(10) = f(2\cdot 5) = f(2) + f(5) = x + y$
   $f(100) = f(10\cdot 10) = f(10) + f(10) = 2(x+y)$
   $f(1000) = f(100\cdot 10) = f(100) + f(10) = \underline{\mathbf{3(x+y)}}$ or $\underline{\mathbf{3x+3y}}$.

2. Parabola: $y^2 - 12x + 24 = 0 \Leftrightarrow x = \frac{1}{12}y^2 + 2$
   $\Rightarrow$ Vertex (2,0), a = 3, opens to the right focus (5,0)

   Circle: $x^2 + y^2 + 8x - 20 = 0 \Leftrightarrow (x+4)^2 + y^2 = 36$
   $\Rightarrow$ Center $(-4,0)$, radius 6
   Thus, the circle passes through the vertex of the parabola
   and, in right ΔTCF, we have $6^2 + FT^2 = 9^2 \Rightarrow FT = \underline{\mathbf{3\sqrt{5}}}$.

   Alternately, using the secant-tangent theorem of geometry,
   $FT^2 = FV(12 + FV) = 3(15) = 45 \Rightarrow FT = \underline{\mathbf{3\sqrt{5}}}$.

3. $\begin{cases} (1)\ x + xy + y = 11 \\ (2)\ x^2y + xy^2 = 30 \end{cases}$

   Let $N = xy$. $(1) \Rightarrow x + y = 11 - N$ and $(2) \Rightarrow xy(x+y) = N(x+y) = 30$
   Thus, $N(11-N) = 30 \Leftrightarrow N^2 - 11N + 30 = (N-5)(N-6) = 0 \Rightarrow N = 5,6$
   $\begin{cases} xy = 5 \\ x + y = 6 \end{cases} \Rightarrow x(6-x) = 5 \Rightarrow x^2 - 6x + 5 = (x-1)(x-5) = 0 \Rightarrow (x,y) = \underline{\mathbf{(1,5),\ (5,1)}}$.
   $\begin{cases} xy = 6 \\ x + y = 5 \end{cases} \Rightarrow x(5-x) = 6 \Rightarrow x^2 - 5x + 6 = (x-2)(x-3) = 0 \Rightarrow (x,y) = \underline{\mathbf{(2,3),\ (3,2)}}$.

Detailed Solutions for GBML Meet 5 - FEBRUARY 2017

ROUND 5

1. $\left(2^{\sin^2 x}\right)\left(2^{\cos^2 x}\right)\left(2^{\tan^2 x}\right)=2^{1+\tan^2 x}=2^{\sec^2 x}=2^4 \Rightarrow \sec x=\pm 2 \Rightarrow \cos x=\pm\frac{1}{2} \Rightarrow x=\underline{-\frac{2\pi}{3},-\frac{\pi}{3},\frac{\pi}{3},\frac{2\pi}{3}}$.

2. $\log_2 3+\log_{36} 3-\log_4 x=\log_{36} 18 \Leftrightarrow \log_2 3-\log_4 x=\log_{36} 18-\log_{36} 3=\log_{36} 6=\frac{1}{2}$

$\Leftrightarrow \log_2 3-\frac{1}{2}\log_2 x=\frac{1}{2}$

$\Leftrightarrow 2\log_2 3-\log_2 x=1$

$\Leftrightarrow \log_2\left(\frac{9}{x}\right)=1$

$\Leftrightarrow \frac{9}{x}=2^1 \Rightarrow x=\underline{\frac{9}{2}}$

3. Applying the definitions of each type of sequence, we have

$$AS:\ x-6=y-x$$
$$GS:\ \frac{y}{x}=\frac{16}{y}$$
$$HS: 16=\frac{2yz}{y+z}$$

Thus, $\begin{cases} y=2x-6 \\ y^2=16x \end{cases} \Rightarrow 4x^2-24x+36=16x \Rightarrow x^2-10x+9=(x-1)(x-9)=0$

$\Rightarrow (x,y)=(1,-4),(9,12)$

Case 1: $(y=-4)$ $8=\frac{-4z}{-4+z} \Rightarrow 2=\frac{-z}{-4+z} \Rightarrow -8+2z=-z \Rightarrow z=\frac{8}{3} \Rightarrow (x,y,z)=\underline{\left(1,-4,\frac{8}{3}\right)}$

Case 2: $(y=12)$ $2=\frac{3z}{12+z} \Rightarrow 24+2z=3z \Rightarrow z=24 \Rightarrow (x,y,z)=\underline{(9,12,24)}$

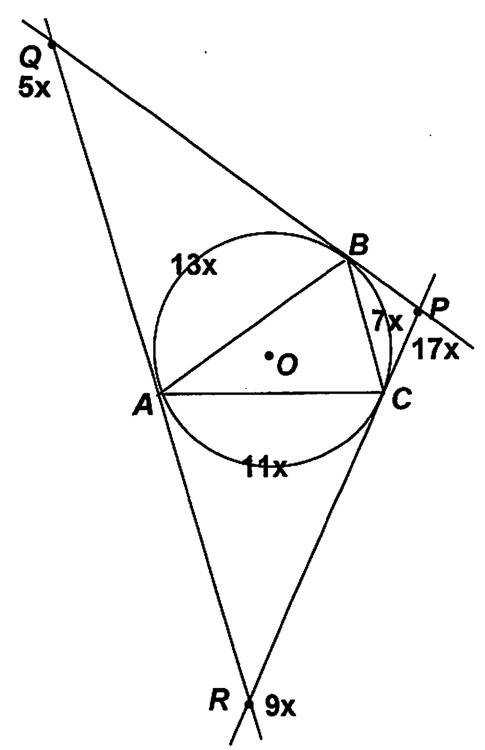
Detailed Solutions for GBML Meet 5 - FEBRUARY 2017

TEAM ROUND

1. The LCM of 2, 3, and 4 is 12. Consider the natural numbers 1, 2, ..., 11, 12. Exactly 5 of them satisfy the divisibility requirements, namely 2, 3, 6, 9, and 10, or the 2^{nd}, 3^{rd}, 6^{th}, 9^{th}, and 10^{th} numbers in the group. Successive groups of 12 natural numbers will have the same distribution of numbers satisfying the divisibility requirements, since successive groups will be of the form $1+12k, 2+12k, 3+12k, ..., 11+12k, 12+12k$, where $k=0$ generated the first group. Adding $12k$ does not change the remainder upon division by 2, 3 or 4. Since $\boxed{99}=19\cdot 5+4$, we are looking in the 20^{th} group of 12 for the 4^{th} number satisfying the divisibility requirements. This will always be the 9^{th} number in that group, or *8 more than the first number*. Thus, the required number is of the form $(1+12k)+8=12k+9$.
   For $k=19$, we have **237**.

   Alternately, once the observation is made that in the set of natural numbers 5 in every 12 consecutive numbers (starting with the group 1, … , 12) will satisfy the stated criteria, it is clear that the last number in the 20^{th} group is $20\cdot 12=240$ and 99^{th} number satisfying the criteria is $240-3=\underline{237}$.

2. Since $m\angle A : m\angle B : m\angle C = 7:11:13$, the corresponding arcs intercepted on circle O are in the same ratio. Call them $7x$, $11x$, and $13x$, respectively. Then: $31x=360°\Rightarrow x=\frac{360}{31}$.

   As angles formed by tangents to circle O, the angles in ΔPQR have measures equal to *half* the difference of the intercepted arcs. Thus,

   $$\begin{cases} m\angle P=((11+13)-7)x/2=17x/2 \\ m\angle Q=((7+11)-13)x/2=5x/2 \\ m\angle R=((7+13)-11)x/2=9x/2 \end{cases}$$

   Since ΔPBC is isosceles,

   $$m\angle PBC=\frac{180-m\angle P}{2}=\frac{180-17\cdot\frac{\mathbf{180}}{31}}{2}=90\left(1-\frac{17}{31}\right)$$

   $$=90\cdot\frac{14}{31}=\frac{1260}{31}=40\frac{20}{31}\Rightarrow \underline{\mathbf{41}}°.$$

TEAM ROUND - continued

3. Given: $\left(x^4+2x^2-3\right)^3\left(x^2-3x+2\right)^4$

The expansion of the first factor produces only even powers of x, plus a constant.
The expansion of the second factor produces both even and odd powers of x, plus a constant.
The highest power of x produced by the first factor is x^{12}; by the second factor, x^8.
x^{15} can be generated in exactly *three* ways.
1) A product of the x^{12} - term from the first expression and x^3 - term from the second.
2) as $x^{10}\cdot x^5$ and, lastly, 3) as $x^8\cdot x^7$
Since each of these products produces *similar* terms, i.e., x^{15}, our task is to find the coefficients of each product and *add* them.

Case 1: $x^{12}\cdot x^3$
The first factor produces $1x^{12}$. Every term in the expansion of the second factor is itself the product of exactly 4 factors chosen from the terms of the second trinomial, namely x^2 , $-3x$ and 2.
x^3 can be produced in only two ways:
Pick x^2 once, $-3x$ once and 2 twice, or don't pick x^2 at all and pick $-3x$ twice and 2 once.
The coefficients in each case can be found using the *multinomial coefficient*, which, for trinomials, is written $\begin{pmatrix} n \\ a\ b\ c\end{pmatrix}$ and evaluated as $\dfrac{n!}{a!\cdot b!\cdot c!}$. a, b, and c represent how many times each term in the trinomial is being used in the product to be determined and $a+b+c=n$.
In this case, the multinomial coefficients are $\begin{pmatrix} 4 \\ 1\,1\,2\end{pmatrix}=\dfrac{4!}{1!1!2!}=12$ and $\begin{pmatrix} 4 \\ 0\,3\,1\end{pmatrix}=\dfrac{4!}{0!\cdot 3!\cdot 1!}=4$.

Thus, the x^3 term is $\begin{pmatrix} 4 \\ 1\,1\,2\end{pmatrix}\left(x^2\right)^1(-3x)^1(2)^2+\begin{pmatrix} 4 \\ 0\,3\,1\end{pmatrix}\left(x^2\right)^0(-3x)^3(2)^1=kx^3$.

$\Rightarrow k=12\cdot 1^1\cdot(-3)^1\cdot(2)^2+4\cdot 1^0\cdot(-3)^3\cdot 2^1=-144-216=-360$. This case gives us $-360x^{15}$.

Case 2: $x^{10}\cdot x^5$ (concentrating on just the numerical components)

The coefficient of x^{10}: $\begin{pmatrix} 3 \\ 2\,1\,0\end{pmatrix}\cdot 1^2\cdot 2^1\cdot(-3)^0=3\cdot 2=6$

The coefficient of x^5: $\begin{pmatrix} 4 \\ 2\,1\,1\end{pmatrix}\cdot 1^2\cdot(-3)^1\cdot 2^1+\begin{pmatrix} 4 \\ 1\,3\,0\end{pmatrix}\cdot 1^1\cdot(-3)^3\cdot 2^0=-72-108=-180$

Thus, this case gives us $-1080x^{15}$.

Case 3: $x^8\cdot x^7$ (concentrating on just the numerical components)

The coefficient of x^8: $\begin{pmatrix} 3 \\ 1\,2\,0\end{pmatrix}\cdot 1^1\cdot 2^2\cdot(-3)^0+\begin{pmatrix} 3 \\ 2\,0\,1\end{pmatrix}\cdot 1^2\cdot 2^0\cdot(-3)^1=12-9=3$

The coefficient of x^7: $\begin{pmatrix} 4 \\ 3\,1\,0\end{pmatrix}\cdot 1^3\cdot(-3)^1 2\cdot^0=-12$. Thus, this case gives us $-36x^{15}$.

The required coefficient is $(-360)+(-1080)+(-36)=\underline{\mathbf{-1476}}$.

Addendum to Question 3 in Round 2:
The sum of three positive numbers is 28, and one is twice another. Two times the largest number is 8 more than the sum of the two smallest numbers. Compute all possible values of the smallest number.

Suppose the sum of the three positive numbers is *some unknown constant k.*
Our expressions for the 3 numbers would then be $a,\ 2a,\ k-3a$.
Case 1: Suppose $2a$ is the largest.
Then the second condition requires that $2(2a)=8+(a+(k-3a)) \Leftrightarrow 4a=8+k-2a \Rightarrow a=\frac{k+8}{6}$

Thus, in terms of k, the three numbers are $\frac{k+8}{6},\ \frac{k+8}{3},\ k-\left(\frac{k+8}{6}+\frac{k+8}{3}\right)=\frac{k-8}{2}$

Since we assumed $2a$ was the largest, it must be true that

$$2a>k-3a \Leftrightarrow \boxed{a>\frac{k}{5}} \Leftrightarrow \frac{k+8}{6}>\frac{k}{5} \Leftrightarrow 5k+40>6k \Rightarrow \underline{\mathbf{k<40}}$$

Case 2: Suppose $k-3a$ is the largest.
Then $2(k-3a)=8+3a \Rightarrow a=\frac{2(k-4)}{9}$ and we have $\frac{2(k-4)}{9}, \frac{4(k-4)}{9}, \frac{k+8}{3}$

$$k-3a>2a \Leftrightarrow \boxed{a<\frac{k}{5}} \Leftrightarrow \frac{2(k-4)}{9}<\frac{k}{5} \Leftrightarrow 10k-40<9k \Leftrightarrow \underline{\mathbf{k<40}}$$

Thus, in any case, the sum of the three numbers must be less than 40.

The original problem had the sum of the three numbers as $k=43$. This gave $a=\frac{17}{2}$ or $\frac{26}{3}$.
You should verify that, although each value certainly produces three numbers which sum to 43, neither of these a-values produces three numbers which satisfy the second condition that "twice the largest will equal 8 plus the sum of the two smaller". There was a hidden inconsistency in the original problem which left students of arithmetic (young and old) a perplexing dilemma. The above algebraic argument shows why the original problem was flawed from the outset.